MATHEMATICS (US)

Paper 0444/23
Paper 23 (Extended)

Key messages

It is important to show the method used in each question in a logical sequence and to set it out neatly in the working space. This is particularly important for questions involving equations which have to be solved for *x*.

General comments

Care must be taken when converting between units, particularly time or area. Time, in particular, cannot be treated as decimals, and the scale factors for area are the squares of the scale factors for length.

Comments on specific questions

Question 1

This question was usually answered correctly. The main error was to find the difference between the numbers and give an answer of 3 (h) 28 (min).

Answer: 2h 32 min

Question 2

This question was answered very well. The only error seen was when the square root of 8 was attempted before adding the 1.

Answer. 3

Question 3

This question was answered very well. The only error seen was when 80 was divided by 4 to reach 20%.

Answer. 5

Question 4

The most usual answer was trapezoid whilst other answers included rhombus and square.

Answer. kite

Question 5

Most candidates gave a correct solution. The most common alternative answer was a partial factorisation such as $3(6x + 9y)$.

Answer. $9(2x+3y)$

Question 6

The correct answers were obtained from the use of the laws of indices. Some saw the root sign and treated it as square root whilst others multiplied the 3 by 2 and gave an answer of 6.

Answer. $\frac{2}{3}$

Question 7

The correct method was to add 55 to 180. Some candidates subtracted 55 from either 100 or 360. Very few candidates drew a diagram which would have assisted them to understand the problem.

Answer. 235

Question 8

There were many correct answers seen. The most common errors were those who had an equivalent answer but not in standard form or those who first added the 3.6 and 5.4 to give 9.

Answer. 5.76×10^{9}

Question 9

Most candidates gained credit for correctly collecting the x's and numbers together. If this resulted in $-10x \geqslant 12$, in many cases $x \geqslant -1.2$ was observed in the answer space. Common errors usually occurred in the manipulation of the inequality and included 19 + 7(= 26) as well as $8x - 2x$. Some candidates left their answer as an unsimplified improper fraction.

Answer: $x \leqslant -1.2$

Question 10

The usual method was to multiply 6000 by 30 to reach 180 000 and many candidates wrote this as their answer. Some tried to convert to square metres by just dividing by 100, giving an answer of 1800.

Answer. 540

Question 11

The most common method was to calculate $\frac{360}{12} = 30$ before subtracting 30 from 180. A less common method was to calculate $(12 - 2) \times 180$ before dividing by 12. The most usual incorrect answer was 30 from a partial method.

Answer: 150

Question 12

Many fully correct solutions were seen and almost all of these multiplied the top equation by 3 and then they added the two resulting equations. Those who attempted to use the method of substitution were usually unsuccessful and some attempted to eliminate a variable without multiplying and they were always unsuccessful.

Answer. [x =] 3 [y =] −2

Question 13

The most common correct method was $\frac{22}{7}-\frac{5}{4}=\frac{88}{28}-\frac{35}{28}=\frac{53}{28}=1\frac{25}{28}$. The most common error was not to write the answer as a mixed number, instead leaving it as the improper fraction $\frac{53}{28}$. The other main error was not to convert the fractions to a common denominator before attempting to subtract them. Some tried to deal with the fractions without involving the integers and occasionally this did lead to the correct answer.

Answer: $1\frac{25}{28}$

Question 14

A very common error seen was the incorrect factorisation of $(x + 4)(5x - 3)$. The answer of $-\frac{5}{3}$ was frequently inaccurate with versions seen including -1.66 and -1.7. Many candidates tried to use the quadratic formula suggesting that they were unable to factorise the expression or that they did not read the question carefully.

Answer: 4, $-\frac{5}{3}$

Question 15

A common error was to write $\left(3+\sqrt{5}\right)^2$ as $3^2 + 5$ and then the rest of the working did not lead to the correct answer.

Answer. $14+8\sqrt{5}$

Question 16

The usual starting point was to draw a perpendicular from point *C* to *AB* and assume that it bisects both angle *C* and line *AB*. Candidates then attempted to use standard trigonometry. The most successful method was to use the sine rule.

Answer. 8

Question 17

The vast majority of candidates gaining full marks wrote their first line as $12m+4xy=xp$. Those candidates who rearranged their first line to be $3m=\frac{xp}{4}-xy$ did not write their second line as $3m=x\left(\frac{p}{4}-xy\right)$. They usually attempted to multiply the equation by 4 and they usually did not multiply the *xy* term by 4.

The other common errors were making a sign error when attempting to rearrange their three term expression, leaving an *x* term on both sides of their equation and choosing one '*x*' term to make the subject of their equation.

Answer: $\frac{12m}{p-4y}$

Question 18

Part (a) was answered correctly by most candidates. In **part (b)** most candidates started to solve the equation $5n^2 + 3 = 608$ by subtracting 3 from 608. However instead of dividing by 5 and then taking the square root, some tried to square root 605. A significant number did not solve this equation but they would state 'yes' and then give an implicit form such as $5 \times 11^2 + 3[= 608]$.

Answer. **(a)** 1, –4 and –9 **(b)** yes because 11 is an integer

Question 19

Many candidates did work out the value of *c* correctly from 12×12. Finding the value of *k* proved to be more challenging. Many gave the value as the radius, 6 or they found the area of the whole circle, thus giving an answer of 36.

Answer. [*k* =]18 [*c* =]144

Question 20

Most candidates struggled to correctly answer this question. In **part (a)(i)** many gave answers of **a**, 2**a** or –2**a** + 2**b**. In **part (a)(ii)** there were many blank responses and many gave answers of –**a** + **b** or just **a**. Very few also answered **part (a)(iii)**. In **part (b)** some made the incorrect assumption that *AT* was parallel to *OB* and therefore they used *AT* as **b** or 2**b**.

Answer. **(a)(i)** $-\mathbf{a} + \mathbf{b}$ **(ii)** $-\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ **(iii)** $\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$ **(b)** $-\frac{1}{2}\mathbf{a} + \frac{3}{2}\mathbf{b}$

Question 21

In **part (a)** the majority of candidates were able to correctly calculate the mean number of hummingbirds seen in Ali's garden. A relatively common error was to misread the graph and calculate the mean number of hummingbirds in Hussein's garden. Some incorrectly multiplied the day number by the number of hummingbirds before adding them together. In **part (b)** some candidates calculated the mean number of hummingbirds seen in one of the two gardens rather than the median for Hussein's garden. Where attempts at listing numbers and finding the median were seen there was a variety of different errors observed, the two most common being working with the data for Ali rather than for Hussein and not ordering the data before choosing the middle one. **Part (c)** was well answered with sometimes 3 or 4 given as the answer.

Answers: **(a)** 3.4 **(b)** 5 **(c)** 10

Question 22

Part (a) was usually answered correctly. The most common incorrect answer was 23°. **Part (b)** was also answered well, the most common error being to assume that both angles *AEB* and *BEC* were 19°. In **part (c)** a good number of candidates gained credit for labelling angle *EAB* as 90°. Incorrect answers were often based on calculations using differing combinations of 19° and 23°, for example 180° – 19° – 23°.

Answers: **(a)** 19 **(b)** 138 **(c)** 90

Question 23

In **part (a)** just a number was required but some candidates wrote it as a cube or as a root. Many however did answer this part correctly. The main error was to include numbers outside the requested range such as 64. In **part (b)** the common incorrect answer was 91. Most answered **part (c)** correctly. The usual incorrect answer in **part (d)** was $\frac{9}{4}$, by omitting the negative power.

Answer. **(a)** 125 or 216 or 343 or 512 or 729 **(b)** 97 **(c)** $7\% < \frac{7}{10} < 0.71 < \sqrt{49}$ **(d)** $\frac{4}{9}$

MATHEMATICS (US)

Paper 0444/43
Paper 43 (Extended)

Key messages

Candidates need to use efficient methods of calculation, show their working and always check their final answers. They should always work with more figures than the final answer requires. In most cases this would need intermediate values written to at least four significant figures.

General comments

This paper gave all candidates an opportunity to demonstrate their knowledge and application of mathematics. Most candidates were able to complete the paper in the allotted time. Few candidates omitted whole questions although some did not attempt several parts. A majority of candidates showed their working and gained method marks but in a significant number of scripts insufficient or no working was seen. In questions requiring candidates to show a result many were unable to gain marks as they used the value they had to show from the beginning. Presentation of work was often good with some scripts showing working that was clearly set out. For less able candidates, working tended to be more haphazard and difficult to follow making it difficult to award method marks when the answer was incorrect. Candidates need to be encouraged to write their working clearly. There were many examples where candidates miscopied their own figures leading to a loss of marks. Similarly, many candidates overwrite their initial answer with a corrected answer. This is often very difficult to read and is not clear what the candidates' final answer is. Candidates should be reminded to re-write rather than overwrite.

Comments on specific questions

Question 1

(a) (i) The question required candidates to show a result and most incorrect answers involved incomplete methods, usually by not showing the division by 10.

(ii) Most candidates used an efficient method, either $12\sin 36$ or $12\cos 54$, although a significant number calculated the third side and then used Pythagoras' theorem. Some calculated the third side and went no further and a small minority could not get started as neither of the angles 36 and 54 had been found. A small number also took the sides to be in the same ratio as the angles.

(b) (i) Many correct answers were seen although some chose to calculate the perimeter and then calculate the longest side from that.

(ii) Although correct answers were seen, candidates were generally less successful in this part as some took the ratio of the angles to be the same as the ratio of the sides leading to a common incorrect answer of 45°.

Answers: **(a)(ii)** 7.05 **(b)(i)** 13 **(ii)** 36.9

Question 2

(a) Almost all candidates gave the correct answer.

(b) (i) Apart from the common errors of 0 and *x*, the vast majority gave the correct answer.

(ii) Almost all candidates gave the correct answer.

(iii) This proved more challenging and far fewer correct answers were seen. Most of the incorrect answers involved errors in dealing with the negative index in the denominator and/or dealing with the coefficient of 3 in the bracket. Common errors involved x^8 in the final answer or coefficients of 3, forgetting to square the coefficient.

(c) (i) Incorrect answers were about as common as fully correct answers. Some candidates earned credit for a partial factorisation such as $(2x + 6)(x - 3)$ and answers such as $2(x^2 - 9)$ were very common.

(ii) Candidates were more successful in this part, with many earning at least 2 marks for a correct factorisation of the denominator. Some of those that only reached $2(x^2 - 9)$ in the previous part were able to complete the factorisation in this part. Many of the less able candidates simply cancelled like terms without any attempt at factorisation.

Answers: **(a)** 343 **(b)(i)** 1 **(ii)** x^{10} **(iii)** $9x^{16}$ **(c)(i)** $2(x - 3)(x + 3)$ **(ii)** $\dfrac{2(x+3)}{x+10}$

Question 3

(a) A majority of candidates appreciated that 456 was 95% of the original price and so divided 456 by 0.95. Some tried, incorrectly, to add 5% of 456, or to find 105% of it. The actual notation wasn't always expressed correctly.

(b) Many candidates were aware of a formula for compound interest but could not quote it correctly. Those that did usually went on to calculate the amount after 18 years. A small number attempted simple interest.

(c) Candidates found this challenging and struggled to make a start. Those that did could find a multiplier for the 17 years but very few could continue to obtain the correct solution.

(d) Very few fully correct solutions were seen. Most candidates forgot to convert p% to a fraction and incorrect answers such as $c - p$, $c - cp$, $c - p\%$ were common.

Answers: **(a)** 480 **(b)** 261.47 **(c)** 1.2 **(d)** $c - \dfrac{cp}{100}$

Question 4

(a) Although some correct intervals were seen, the most common error was giving the middle interval $70 < t \leqslant 80$.

(b) A majority of candidates appeared not to understand the process for finding the mean of grouped data and made no meaningful progress. Those that did understand usually obtained full marks. A significant proportion made no attempt.

(c) (i) Some stated that the intervals were unequal. Others stated that the individual times were unknown but did not specifically relate this to the highest and lowest times. Only a minority gave a complete explanation.

(ii) If a candidate knew what to do they usually went on to earn full credit. Many used the efficient method, $\dfrac{26}{150} \times 360$, but it was quite common to see candidates converting $\dfrac{26}{150}$ to a percentage, usually 17.3%, but this then lost the final mark as the answer was often inaccurate. Others lost marks by using the wrong intervals and 10 and 34 were sometimes used instead of 26.

(d) A majority of candidates gave the correct answer.

(e) (i) Few candidates were able to cope with two events but some managed to write down a correct product. One incorrect answer was $\frac{1}{225}$ from replacement. Some forgot to include the probability for the second girl chosen at random. Several with an incorrect answer showed no working and could not be awarded any method marks.

(ii) This proved more challenging than the previous part and correct answers were rare. Most attempts that showed any working were treating the question as though it was probability with replacement. In addition candidates forgot that there were two combinations that would satisfy the given conditions. Some added their probabilities rather than multiplying.

(f) Candidates seemed unfamiliar with this style of question on the histogram and fully correct answers were rare. The one height that was given happened to be half of the frequency for that interval as well as being the width of the interval. Many of the incorrect responses followed one of these two patterns. Evidence of any working was rare but some candidates did list the frequency densities as their heights.

Answers: **(a)** $80 < t \leqslant 100$ **(b)** 86 **(c)(ii)** 62.4 **(d)** $\frac{22}{150}$ **(e)(i)** $\frac{90}{22350}$ **(ii)** $\frac{440}{22350}$ **(f)** 13, 8.5, 7.25, 1.1

Question 5

(a) (i) Many correct answers were seen. Reflection in the *x*-axis was the most common incorrect answer.

(ii) Candidates were less successful with the enlargement. Many with an incorrect answer earned partial credit for an enlargement of the correct size and orientation but in the wrong position. Many of those with the wrong position had the apex of the triangle at (0, 4).

(iii) A majority of candidates drew the correct image for the translation. Some earned partial credit for a translation with a correct displacement in one direction. Several candidates treated the translation as $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

(b) Fully correct or partially correct answers were in the minority. The centre of rotation caused the most difficulty. Some that gave the answer as rotation could not identify the correct centre and often forgot the amount and direction of the rotation.

Answers: **(b)** Rotation, 90° clockwise, (4, –1)

Question 6

(a) A large majority of correct answers were seen.

(b) A majority of candidates understood what was required and earned full marks. Some got as far as $2(3 - x) - 1$ but then expanded the bracket incorrectly as $6 - x$. Some expanded correctly but did not simplify the resulting expression.

(c) (i) Many were able to set up and solve the resulting equation. Collecting the terms gave rise to the most errors.

(ii) Only a minority coped with finding the value of *x*. The most common errors usually involved calculating $2^{0.125}$ and 2×0.125.

(d) Many candidates seemed familiar with the process of finding an inverse function and this was reflected in the number of correct responses. Common errors seen included $\frac{1}{2x-1}$ and $-(2x - 1)$.

(e) A small majority were able to give the first correct step but few could go on to give a correct expression in its simplest form. Incorrect answers seen included $\frac{2}{3-x}$ and $\frac{2g}{x}$.

(f) This proved challenging for all but the most able candidates. By far the most common incorrect answer was 2 and others included answers based on powers of 2. Some reached 2^4 but then evaluated it as 8.

Answers: **(a)** –7 **(b)** $5 - 2x$ **(c)(i)** $\frac{4}{3}$ **(ii)** –3 **(d)** $\frac{x+1}{2}$ **(e)** $\frac{3x-2}{x}$ **(f)** 16

Question 7

(a) (i) Some candidates gave solutions with working clearly laid out. Others showed the correct method but the accuracy of their final answer was outside of the acceptable range. The two most common errors involved using 10 as the radius instead of 5 and using $2\pi r^2 h$ for the volume of a cylinder.

(ii) Fully correct solutions were rarely seen as many candidates used the wrong formula for a hemisphere, usually using the formula for a whole sphere instead. If the equation was set up correctly at the start, the radius was usually calculated correctly. Some lost the final mark by using square root instead of cube root, despite setting out their working correctly.

(iii) Some candidates showed the correct working and were able to give the correct answer. A few candidates did not earn full marks, giving the area of one face only instead of the total surface area. Others lost the final mark by not maintaining sufficient accuracy, usually by giving the cube root of 2000 to only three figures. Other common errors involved multiplication by 2 when trying to square the length of a side and division by 3 when trying to find the cube root.

(b) (i) Roughly half of all candidates made a successful attempt at the question. However, several attempted to calculate the perpendicular height but then rarely went any further. Some used cosine in the area formula instead of sine.

(ii) Few candidates demonstrated competency in identifying and using the cosine rule. Those that did were able to find the correct length for the third side and show a correct method for finding the perimeter. This was expected to be 23.46 or better but premature rounding of the third side resulted in a significant number of candidates only giving the perimeter as 23.5. Some candidates attempted less efficient methods, drawing the perpendicular height and attempting to work their way through two right-angled triangles. In most cases the method was never completed. Many applied a circular argument, calculating the length of the third side from the given perimeter and then adding the third side to 7 and 10. This earned no credit.

(c) This proved more challenging and few fully correct answers were seen. Many candidates used the 28.2 as the arc length earning only partial credit; others used the formula for area instead of arc length. A few used 10.2 as the third side of the triangle and used the cosine rule to calculate the angle *l*.

Answers: **(a)(i)** 25.5 **(ii)** 9.85 **(iii)** 952 **(b)(i)** 22.5 **(c)** 64.9

Question 8

(a) Most candidates completed the table correctly with many of the errors arising from incorrect squaring of negative values of *x*.

(b) The plotting of the points was carried out accurately with many going on to draw a smooth curve. When points were plotted correctly any loss of marks was the result of the curve missing one or more points, the curve having straight line segments or occasionally excessive feathering.

(c) Candidates understood what was required and were able to read off at least one of the values accurately. For some, reading the scale led to errors.

(d) This proved challenging for all but the most able candidates. Those that attempted to complete the square often started incorrectly with $(x + 2.5)^2$. Others attempted to expand $2(x + a)^2 + b$ with the intention of equating coefficients but few were successful. Common errors tended to lead to terms such as $2ax$, a^2 and $2b$.

Answers: **(a)** 9, –6, 9 **(c)** –3.4, 0.9 **(d)** $a = \frac{5}{4}$, $b = -\frac{49}{8}$

Question 9

(a) (i) Many correct answers were seen.

(ii) A majority of candidates gave the correct gradient but many didn't appreciate that the equation should be written in the form $y = mx + b$. Apart from the incorrect answer of $\frac{3}{2}$, other common errors included 3, –3, $\frac{2}{3}$ and $-\frac{2}{3}$.

(b) A small majority gave a correct point on the x-axis. A wide variety of incorrect answers were given, frequently without any working. The most common errors included (0, 0.8) and the intercept with the y-axis.

(c) The process of finding the gradient of a perpendicular line was not understood by many of the candidates and only a minority gave a correct equation. Many of the incorrect equations resulted from an incorrect gradient, often given as 5, –5, $\frac{1}{5}$ and a variety of others. With or without the correct gradient, not all candidates appreciated that (10, 9) needed to be substituted to find the constant term. Some made no attempt.

(d) This proved quite challenging and a large minority of candidates gave fully correct answers. Those that found the co-ordinates either solved the simultaneous equations algebraically or drew sketches of the two lines. Some attempted to find the point of intersection of line A and its perpendicular line. Many gave an incorrect pair of co-ordinates without showing any working and were unable to earn any method marks.

(e) In addition to a very high proportion of candidates that made no attempt, a majority of those that made an attempt struggled to make any progress. Many of those that were successful had drawn a diagram showing the triangle with intercepts clearly labelled.

Answers: **(a)(i)** 5 **(ii)** $-\frac{3}{2}$ **(b)** (0.8, 0) **(c)** $y = -0.2x + 11$ **(d)** (2, 6) **(e)** 13

Question 10

(a) More able candidates were able to give the correct expression for the time. Common errors included $\frac{10}{x} - 0.5$ and $\frac{10}{x + 0.5}$.

(b) (i) Those candidates that had struggled in the previous part struggled again or made no attempt. When setting up an equation it was common to see the times subtracted in the wrong order, and occasionally added. Algebraic fractions were often successfully combined but the expansion of brackets often led to errors with the signs. Some tried unsuccessfully to work backwards from the given equation, or use values found in the next part. Less able candidates often attempted to solve the quadratic equation.

(ii) Some candidates made a good attempt and usually obtained two correct solutions, although not always written to 2 decimal places as requested. When solving the equation some candidates made sign errors and the use of –1 instead of –(–1) for $-b$ and squaring –1 to obtain –1 were common errors.

(iii) Many candidates made no attempt to find the time. Those attempting the question usually earned some credit but only a minority gave a fully correct solution. Some correctly calculated the time in hours but did not go on to give their answer in hours and minutes. Others that did convert their answers sometimes forgot to round their answer to the nearest minute. Some attempted to calculate Alfredo's time instead of Luigi's.

Answers: **(a)** $\dfrac{10}{x-0.5}$ **(b)(ii)** –4.23 and 4.73 **(iii)** 2 h 7 min

Question 11

(a) (i) Many candidates were successful in finding the prime factors, often by using a factor tree. Occasionally not all factors were reduced to primes but, in a few cases, some credit was earned. Some gave an answer as the sum of prime factors rather than as a product.

(ii) The majority were able to give the correct LCM, the most common method being to list the multiples of both numbers. Some gave 540 without any working. A common error was to think this was GCF giving an answer of 18 or to give a higher multiple of both which was usually 9720.

(b) This proved challenging for the majority of candidates and a very high proportion made no attempt. Many of the candidates could not see the link between the given expressions for *X* and *Y* and the LCM and GCF values resulting in no progress being made. Others started by attempting to find prime factor products for 1225 and 42 875, but did not make enough progress to deduce that *a* = 5.

Answers: **(a)(i)** $2^2 \times 3^2 \times 5$ **(ii)** 540 **(b)** $X = 8575$, $Y = 6125$